DISSOLUTION OF A HOT BODY IN CONTACT WITH A FREE LIQUID SURFACE

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The effect of capillary forces on the dissolution of a hot body in contact with a free liquid surface is examined.

The surface tension α depends on the temperature T and concentration c of the solution [1], so that for small temperature intervals ΔT and low concentrations we can write approximately

$$\alpha = \alpha_0 + \alpha_1 \Delta T + \alpha_2 c. \tag{1}$$

The constant coefficient $\alpha_1 = d\alpha/dT < 0$ for all liquids. As for the coefficient α_2, it may be either negative or positive. We consider the case $\alpha_2 > 0$. The dependence of surface tension on temperature and concentration gives the result that the hot and saturated layers of solution directly adjacent to the dissolving body spread over the surface of the liquid under the action of surface tension forces. This motion is called capillary convection [2].

Capillary convection leads to intense dissolution not only in thin layers of liquid but also in volumes whose dimensions are of the same order in all directions [3].

Below it is shown how surface tension affects the steady dissolution of hot bodies at the surface of a liquid. An exact solution of the problem is found in the absence of a gravitational field. Graphs of the lines of equal concentration and streamlines are presented.

We note that in the more general formulation, with allowance for gravitational convection, the problem does not have an exact solution. On the other hand, it is clear that under certain conditions (light layers at the top) gravitational convection will play no part. This occurs, for example, when the density of the hot body is low in comparison with the liquid.

Consider a liquid occupying the half-space $z > 0$. The free surface of the liquid is at $z = 0$. At the origin of the spherical coordinate system a heated rod is advanced into the liquid at a constant rate v_0, such that the end of the rod is able to dissolve without penetrating into the interior of the liquid. If we denote the cross section of the rod by S_0 and its density by ρ_0, in unit time $I = v_0\rho_0 S_0$ units of mass are dissolved. The cross section of the rod is assumed to be small and is not taken into account in the problem. The rod supplies Q units of heat to the liquid each second.

In the absence of a gravitational field, the Navier-Stokes equations for steady free convection, heat conduction, and diffusion and the continuity equation have the form

$$\begin{gathered}(\mathbf{v}\,\Delta)\,\mathbf{v} = -\frac{1}{\rho}\,\nabla p + \nu\nabla^2\mathbf{v},\\ \mathbf{v}\nabla T = \chi\nabla^2 T,\\ \mathbf{v}\nabla c = D\,\nabla^2 c,\\ \operatorname{div}\mathbf{v} = 0.\end{gathered} \tag{2}$$

From the symmetry of the problem, it follows that

$$v_\varphi = \frac{\partial v}{\partial \varphi} = 0. \tag{3}$$

Moreover, from the symmetry conditions on the z-axis ($\vartheta = 0$) we can write

$$\frac{\partial v_r}{\partial \vartheta} = \frac{\partial T}{\partial \vartheta} = \frac{\partial p}{\partial \vartheta} = \frac{\partial c}{\partial \vartheta} = v_\vartheta = 0. \tag{4}$$

At the liquid surface ($\vartheta = \pi/2$) there is no normal velocity component and no heat or diffusion fluxes across the surface:

$$v_\vartheta = \frac{\partial T}{\partial \vartheta} = \frac{\partial c}{\partial \vartheta} = 0. \tag{5}$$

Furthermore, at the liquid surface the tangential component of the viscous stress tensor must be balanced by the tangential stress due to the variation of α (see (1)) along the surface [2,3]:

$$\eta\,\frac{1}{r}\,\frac{\partial v_r}{\partial \vartheta} = \alpha_1\,\frac{\partial T}{\partial r} + \alpha_2\,\frac{\partial c}{\partial r}\,. \tag{6}$$

At infinity, all the functions must approach zero. And, finally, the following integral conditions of constancy of the mass and heat flows through a hemisphere of any radius with center at the coordinate origin must be satisfied:

$$\begin{gathered}2\pi\rho\int\limits_0^{\pi/2}\left(cv_r - D\,\frac{\partial c}{\partial r}\right) r^2\sin\vartheta\, d\vartheta = I,\\ 2\pi\rho\, c_p\int\limits_0^{\pi/2}\left(Tv_r - \chi\,\frac{\partial T}{\partial r}\right) r^2\sin\vartheta\, d\vartheta = Q.\end{gathered} \tag{7}$$

We emphasize that the quantity I is arbitrary; i.e., there are no limitations on the actual rate of solution.

System (2) admits the separation of variables:

$$\begin{gathered}v_r = \nu\,\frac{\theta_1(\vartheta)}{r},\quad v_\vartheta = \nu\,\frac{\theta_2(\vartheta)}{r},\\ T = \frac{\eta\nu}{\alpha_1}\,\frac{\theta_3(\vartheta)}{r},\quad c = \frac{\eta\nu}{\alpha_2}\,\frac{\theta_4(\vartheta)}{r}\,.\end{gathered} \tag{8}$$

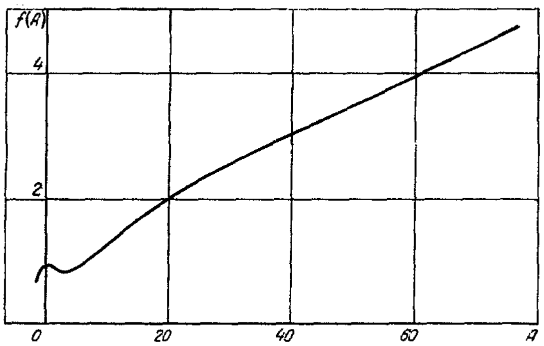

Fig. 1. f(A) as a function of A.

This form of the functions automatically satisfies the condition of subsidence at infinity.

Substituting functions (8) into (2) and carrying out the necessary operations (see [3]), using (3), (4), and (5), we obtain

$$\theta_2 = -2\,\frac{d\ln w}{d\vartheta},\quad \theta_1 = -\frac{(\sin\vartheta\theta_2)'}{\sin\vartheta},$$
$$\theta_3 = (1+A)^{\mathrm{P}}\frac{C_1}{w^{2\mathrm{P}}},\quad \theta_4 = (1+A)^{\mathrm{S}}\frac{C_2}{w^{2\mathrm{S}}}, \tag{9}$$
$$w = n_2(1+\cos\vartheta)^{n_1} - n_1(1+\cos\vartheta)^{n_2};$$
$$n_{1,2} = \frac{1\pm\varepsilon}{2};\quad \varepsilon = \sqrt{1+A};\ \mathrm{P} = \frac{\nu}{\chi};\quad \mathrm{S} = \frac{\nu}{D}.$$

The constants A, C_1, and C_2 are found from conditions (6) and (7). In the notation

$$R_T = \frac{Q}{2\pi}\,\frac{a_1}{\varkappa\nu\eta},\quad R_C = \frac{I}{2\pi}\,\frac{a_2}{D\eta^2} \tag{10}$$

these equations take the form

$$A = C_1 + C_2,$$
$$\int_0^1 (1+\mathrm{P}^2\theta_2^2)\,\theta_3\,dt = R_T, \tag{11}$$
$$\int_0^1 (1+\mathrm{S}^2\theta_2^2)\,\theta_4\,dt = R_C.$$

System (11) gives A, C_1, and C_2 as functions of the four parameters R_T, R_C, P, and S and is too complicated for a review of the results, in the general case. Below we consider the special case P = S = 1. Then A, C_1, and C_2 are found from the equations

$$A = C_1 + C_2,$$
$$C_1 = \frac{R_T}{f(A)},\quad C_2 = \frac{R_C}{f(A)}, \tag{12}$$
$$f(A) = (1+A)\left\{\int_0^1 \frac{dt}{w^2} + \int_0^1 \frac{\theta_2^2\,dt}{w^2}\right\}.$$

The first integral in f(A) is evaluated exactly (it determines the diffusion heat and mass transfer):

$$\int_0^1 \frac{dt}{w^2} = \frac{1}{n_2\varepsilon}\left[\frac{1}{n_2-n_1} - \frac{1}{n_2 2^{\varepsilon} - n_1}\right].$$

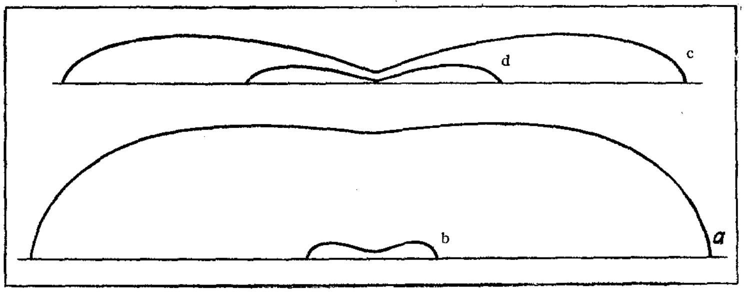

Fig. 2. Equal-concentration lines: a) R_T = 0, R_C = 9; b) 41 and 9; c) 0 and 160; d) 277 and 160.

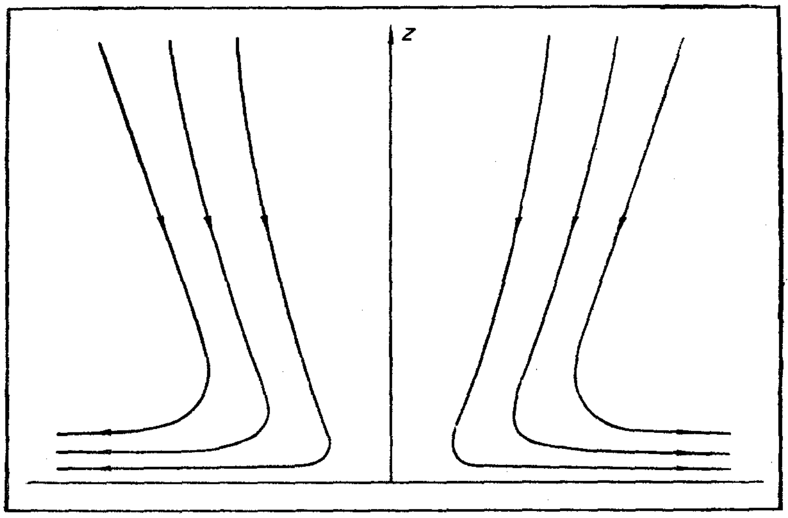

Fig. 3. Streamlines (A = 24).

The second integral in $f(A)$ determines the convective transfer. At large R_T and R_C, this integral is much larger than the first. The function $f(A)$ from (12) is represented in Fig. 1.

The results of the calculations are reproduced in Figs. 2 and 3. Figure 2 shows the lines of equal concentration (the isotherms look exactly the same). Curve a corresponds to an isothermal liquid and small mass fluxes. The form of the isoline, which closely resembles a semicircle, indicates the predominance of the diffusion mass transfer mechanism. Curve b is the same line of equal concentration at relatively low heat fluxes. The sharp compression of the isolines points to developed thermocapillary convection, which entrains the dense layers of solution and carries them along the surface of the liquid. The curve c represents the line of equal concentration in the isothermal liquid at large mass fluxes. Curve d represents the same isoline at large R_T.

The streamlines are shown in Fig. 3. The direction of motion is indicated by the arrows.

In Fig. 2 (curves b, c, d) and Fig. 3, the capillary convection mechanism is plainly visible. The capillary forces, directed toward the cold layers and low concentrations, stretch the isolines along the surface, while the liquid flowing along the z-axis from the interior pinches the isolines at the center.

It is not difficult to extend these results to the dissolving of a cold body in a hot liquid, to the case of negative sorption, etc.

It is interesting to note that when P = S the terms R_T and R_C enter into the equations in exactly the same way, so that when $R_T = -R_C$ (cold rod), capillary convection does not occur.

NOTATION

α is the surface tension; z is the polar axis; v_0, S_0, and ρ_0 are the velocity, cross section, and density of the rod; I is the mass flux; Q is the heat flux; $v(v_r, v_\vartheta, v_\varphi)$ is the velocity of the liquid; p is the pressure; T is the temperature; c is the concentration; ρ is the density; χ is the thermal diffusivity; D is the diffusion coefficient; η is the dynamic viscosity; ν is the kinematic viscosity; r, ϑ, and φ are spherical coordinates; P is the Prandtl number; S is the Schmidt number (diffusion Prandtl number); R_C is the dimensionless mass flux; R_T is the dimensionless heat flux.

REFERENCES

1. L. D. Landau and E. M. Lifshits, The Mechanics of Continuous Media [in Russian], Gostekhizdat, 1954.

2. V. G. Levich, Physicochemical Hydrodynamics [in Russian], Fizmatgiz, 1959.

3. Yu. K. Bratukhin and L. N. Maurin, Prikladnaya matematika i mekhanika, **31**, 577, 1967.

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